Published by Institute of Physics Publishing for SISSA/ISAS

Received: *January 6, 2003*
Revised: *February 27, 2003*
Accepted: *March 9, 2003*

Three-form flux with $\mathcal{N} = 2$ supersymmetry on $\mathrm{AdS}_5 \times \mathrm{S}^5$

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Abstract: In the context of the AdS/CFT correspondence the general form of a three-form flux perturbation to the $\mathrm{AdS}_5\times \mathrm{S}^5$ solution in the type-IIB supergravity which preserves $\mathcal{N} = 2$ supersymmetry is obtained. The arbitrary holomorphic function appearing in the $\mathcal{N} = 1$ case studied by Graña and Polchinski is restricted to a quadratic function of the coordinates transverse to the D3-branes.

Keywords: AdS-CFT and dS-CFT Correspondence, Supersymmetry and Duality, Supergravity Models.

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1. Introduction

It was proposed that the type-IIB string theory compactified on $\mathrm{AdS}_5 \times \mathrm{S}^5$ has a dual description by the $\mathcal{N}=4$ super Yang-Mills theory in the large N limit [1]–[3]. This conjecture of the AdS/CFT correspondence has been supported by comparison of spectra, correlation functions and anomalies calculated in both of the supergravity and the Yang-Mills theory. (For a review, see ref. [4].) The AdS/CFT correspondence was also studied in various other spacetime dimensions. At first the correspondence was studied for theories with high supersymmetries such as $\mathcal{N}=4$. To apply it to more realistic models one has to consider theories with lower supersymmtries.

One of the ways to obtain the AdS/CFT correspondence for lower supersymmetric cases is to modify supergravity solutions by adding a perturbation. In ref. [5] a perturbation of the three-form flux was added to the $\mathrm{AdS}_5 \times \mathrm{S}^5$, which breaks $\mathcal{N}=4$ to $\mathcal{N}=1$. This perturbation corresponds to fermion mass terms of the three $\mathcal{N}=1$ chiral multiplets in the $\mathcal{N}=4$ super Yang-Mills theory and polarizes D3 branes into 5-branes [6, 7]. Similar constructions of the AdS/CFT correspondence with lower supersymmetries were discussed in refs. [8]–[11].

The general form of a three-form flux perturbation to the $\mathrm{AdS}_5 \times \mathrm{S}^5$ solution which preserves $\mathcal{N}=1$ supersymmetry and satisfies the Bianchi identity and the linearized field equation was obtained in ref. [12]. It contains an arbitrary holomorphic function and an arbitrary harmonic function of the coordinates for the directions transverse to the D3-branes. It was argued that the holomorphic function corresponds to a superpotential in the dual super Yang-Mills theory. When the holomorphic function is quadratic in the transverse coordinates, the three-form flux coincides with that of ref. [5].

The purpose of the present paper is to obtain the general form of a three-form flux perturbation to the $\mathrm{AdS}_5 \times \mathrm{S}^5$ solution which preserves $\mathcal{N}=2$ supersymmetry. We use the result of the $\mathcal{N}=1$ case [12] and require further that the second supersymmetry is preserved. We find that the arbitrary holomorphic function in the $\mathcal{N}=1$ case is restricted to a quadratic function of the transverse coordinates. This is a special form of the perturbation studied in ref. [5], which has one vanishing mass.

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It would be interesting to study a relation of our result to other works on soft breaking of $\mathcal{N}=4$ to $\mathcal{N}=2$ in the Coulomb branch [13]–[15]. In order to discuss the corresponding dual field theory and its RG flows we need to find out an exact solution with non-vanishing three-form flux. In addition, it would be also interesting to discuss the brane representations and massive vacua using S-dual transformations.

2. Unperturbed solution

The field content of the type-IIB supergravity in ten dimensions [16, 17] is a metric g_{MN}, a complex Rarita-Schwinger field ψ_M, a real fourth-rank antisymmetric tensor field with a self-dual field strength F_{MNPQR}, a complex second-rank antisymmetric tensor field with a field strength G_{MNP}, a complex spinor field λ and a complex scalar filed $\tau = C + ie^{-\Phi}$. We denote ten-dimensional world indices as $M, N, \ldots = 0, 1, \ldots, 9$ and local Lorentz indices as $A, B, \ldots = 0, 1, \ldots, 9$. The fermionic fields satisfy chirality conditions $\bar{\Gamma}_{10D}\psi_M = \psi_M$, $\bar{\Gamma}_{10D}\lambda = -\lambda$, where $\bar{\Gamma}_{10D} = \Gamma^0\Gamma^1\cdots\Gamma^9$ is the ten-dimensional chirality matrix. We choose the ten-dimensional gamma matrices Γ^A to have real components.

The field equations of this theory have a solution with the $\mathrm{AdS}_5 \times \mathrm{S}^5$ metric [18, 19]

$$g_{MN}dx^M dx^N = Z^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + Z^{1/2}\delta_{mn}dx^m dx^n\,, \tag{2.1}$$

where $M = (\mu, m)$ ($\mu = 0, 1, 2, 3;\ m = 4, 5, \ldots, 9$), $Z = R^4/r^4$ and $r^2 = x^m x^n \delta_{mn}$. The constant R is a radius of AdS_5 and S^5. The fifth-rank field strength has non-vanishing components

$$\begin{aligned} F_{\mu\nu\rho\sigma m} &= \frac{1}{\kappa Z^2}\epsilon_{\mu\nu\rho\sigma}\partial_m Z\,, \\ F_{mnpqr} &= -\frac{Z^{1/2}}{\kappa}\epsilon_{mnpqrs}\partial^s Z\,, \end{aligned} \tag{2.2}$$

where κ is a coupling constant. This solution represents a supergravity configuration produced by D3-branes located at $x^m = 0$. More generally, the warp factor Z can be an arbitrary function of x^m which is harmonic except at points where D3-branes exist. We will consider the general Z but assume that the density of D3-branes vanishes for $r \to \infty$ and therefore $Z \to R^4/r^4$ for $r \to \infty$.

We are interested in how many supersymmetries are preserved by this solution and by a solution with a perturbation of G_{MNP} discussed later. They are found by studying vanishing of local supertransformations of the fermionic fields ψ_M and λ. The supertransformations of the fermionic fields [16, 17] in these backgrounds are

$$\begin{aligned} \delta\psi_M &= \frac{1}{\kappa}D_M\epsilon + \frac{1}{16\cdot 5!}iF_{P_1\cdots P_5}\Gamma^{P_1\cdots P_5}\Gamma_M\epsilon - \frac{1}{96}G_{NPQ}\left(\Gamma_M{}^{NPQ} - 9\delta_M^N\Gamma^{PQ}\right)\epsilon^*\,, \\ \delta\lambda &= \frac{1}{24}G_{MNP}\Gamma^{MNP}\epsilon\,, \end{aligned} \tag{2.3}$$

where the transformation parameter ϵ is a complex spinor satisfying the chirality condition $\bar{\Gamma}_{10D}\epsilon = \epsilon$. To study the supertransformations for the above backgrounds it is convenient

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to represent the ten-dimensional gamma matrices as

$$\begin{aligned}\Gamma^{\mu} &= \gamma^{\mu} \otimes \mathbf{1}\,,\\ \Gamma^{m} &= \bar{\gamma}_{4D} \otimes \gamma^{m}\,,\end{aligned} \tag{2.4}$$

where γ^{μ} and γ^{m} are the SO(3,1) and SO(6) gamma matrices respectively. The chirality matrices are defined as

$$\bar{\gamma}_{4D} = i\gamma^0\gamma^1\gamma^2\gamma^3\,, \qquad \bar{\gamma}_{6D} = i\gamma^4\gamma^5\gamma^6\gamma^7\gamma^8\gamma^9\,, \tag{2.5}$$

which are related to the ten-dimensional one as $\bar{\Gamma}_{10D} = -\bar{\gamma}_{4D}\bar{\gamma}_{6D}$.

The above solution (2.1) and (2.2) without a perturbation has 32 supersymmetries [18, 19]. This can be seen as follows. The supertransformation $\delta\lambda$ automatically vanishes, while the vanishing of $\delta\psi_M$ requires

$$\tilde{D}_M \epsilon = 0\,, \tag{2.6}$$

where we have defined

$$\begin{aligned}\tilde{D}_{\mu} &= \partial_{\mu} - \frac{1}{8Z}\partial_m Z \gamma_{\mu}\gamma^m (1+\bar{\gamma}_{4D})\,,\\ \tilde{D}_m &= \partial_m - \frac{1}{8Z}\partial_n Z\,(\delta_m^n \bar{\gamma}_{4D} - \gamma_m{}^n(1+\bar{\gamma}_{4D}))\,.\end{aligned} \tag{2.7}$$

For solutions of eq. (2.6) to exist the integrability condition

$$[\tilde{D}_M, \tilde{D}_N]\epsilon = 0 \tag{2.8}$$

must be satisfied. Using the expression (2.7) it is easy to show that eq. (2.8) is satisfied for an arbitrary ϵ. Therefore, all of 32 supersymmetries are preserved [18, 19]. From the four-dimensional field theoretical point of view in the AdS/CFT correspondence 16 of them are Poincaré supersymmetries while other 16 are conformal supersymmetries. Thus, we have $\mathcal{N} = 4$ supersymmetry in four dimensions.

More explicitly, the solutions of eq. (2.6) with the chirality $\bar{\gamma}_{4D} = -1$ have a form

$$\epsilon = Z^{-1/8}\eta\,, \tag{2.9}$$

where η is an arbitrary constant spinor with the chirality $\bar{\gamma}_{4D} = -1$. These solutions correspond to Poincaré supersymmetries. The solutions with the chirality $\bar{\gamma}_{4D} = +1$ depend on x^{μ} and correspond to conformal supersymmetries.

3. Three-form flux with $\mathcal{N} = 2$ supersymmetry

By introducing a perturbation of the three-form flux G_{mnp} the $\mathcal{N} = 4$ supersymmetry of the unperturbed supergravity background is broken to lower $\mathcal{N}$. In ref. [12] the conditions on G_{mnp} for unbroken $\mathcal{N} = 1$ supersymmetry were studied. The supersymmetry parameter is expanded as $\epsilon = \epsilon_0 + \epsilon_1 + \cdots$, where ϵ_0 is a solution of eq. (2.6) for the unperturbed background and ϵ_1 is the first order correction due to the perturbation. Substituting it into eq. (2.6) ϵ_1 is determined by ϵ_0. To proceed it is convenient to define complex coordinates z^i $(i = 1, 2, 3)$ from x^m

$$z^1 = \frac{1}{\sqrt{2}}(x^4 + ix^7)\,, \qquad z^2 = \frac{1}{\sqrt{2}}(x^5 + ix^8)\,, \qquad z^3 = \frac{1}{\sqrt{2}}(x^6 + ix^9)\,. \tag{3.1}$$

It was required in ref. [12] that one of the four Poincaré supersymmetries $\epsilon_0 = Z^{-1/8}\eta$, where η is a constant spinor satisfying

$$\gamma^{\bar{1}}\eta = \gamma^{\bar{2}}\eta = \gamma^{\bar{3}}\eta = 0\,, \tag{3.2}$$

is preserved. Here, $\bar{i}$ denote indices of $\bar{z}^i$, while i denote those of z^i. Using the expression $\bar{\gamma}_{6D} = (1 - \gamma^1\gamma^{\bar{1}})(1 - \gamma^2\gamma^{\bar{2}})(1 - \gamma^3\gamma^{\bar{3}})$ it is easy to see that this ϵ_0 has the chirality $\bar{\gamma}_{4D} = -\bar{\gamma}_{6D} = -1$ appropriate for the Poincaré supersymmetry. Then, this $\mathcal{N} = 1$ supersymmetry restricts the form of G_{mnp} as [12]

$$\begin{aligned}
G_{ijk} &= 0\,, \\
G_{ij\bar{k}} &= \frac{2}{3}\hat{\epsilon}_{\bar{k}}{}^{pq}\partial^{-2}\partial_p\partial_{[i}\phi\partial_{j]}\partial_q Z + \hat{\epsilon}_{ij}{}^{\bar{l}}\partial_{\bar{k}}\partial_{\bar{l}}\psi\,, \\
G_{i\bar{j}\bar{k}} &= \frac{1}{12}\hat{\epsilon}_{\bar{j}\bar{k}}{}^{l}\left(2\partial_i\partial_l\phi Z - \alpha\hat{\epsilon}_{il}{}^{\bar{k}}\partial_{\bar{k}}Z - 4\partial_{[i}\phi\partial_{l]}Z\right), \\
G_{\bar{i}\bar{j}\bar{k}} &= \frac{1}{6}\hat{\epsilon}_{\bar{i}\bar{j}\bar{k}}\delta^{l\bar{l}}\partial_l\phi\partial_{\bar{l}}Z\,,
\end{aligned} \tag{3.3}$$

where $\phi(z^1, z^2, z^3)$ is an arbitrary holomorphic function, α is an arbitrary constant and ψ is an arbitrary harmonic function.[1] In eq. (3.3) $\hat{\epsilon}_{ij}{}^{\bar{k}}$ and $\hat{\epsilon}_{\bar{i}\bar{j}}{}^{k}$ are totally antisymmetric in their indices and take constant values 0, ± 1 regardless of index positions, and $\partial^2 = 2\delta^{i\bar{i}}\partial_i\partial_{\bar{i}}$ is the laplacian. The three-form flux (3.3) also satisfies the Bianchi identity as well as the linearized field equation.

We shall obtain conditions on G_{mnp} for unbroken $\mathcal{N} = 2$ supersymmetry. We require that in addition to $\epsilon_0 = Z^{-1/8}\eta$ the second supersymmetry with the parameter

$$\epsilon_0 = Z^{-1/8}\gamma^1\gamma^2\eta \tag{3.4}$$

is also preserved. This ϵ_0 satisfies

$$\gamma^1\epsilon_0 = \gamma^2\epsilon_0 = \gamma^{\bar{3}}\epsilon_0 = 0 \tag{3.5}$$

and has the chirality $\bar{\gamma}_{4D} = -1$. Comparing eqs. (3.2) and (3.5) it is easy to see that the conditions for the second supersymmetry are obtained from eq. (3.3) by the replacements

$$1 \leftrightarrow \bar{1}\,, \qquad 2 \leftrightarrow \bar{2}\,, \qquad \alpha \to \alpha'\,, \qquad \phi(z^1, z^2, z^3) \to \phi'(\bar{z}^1, \bar{z}^2, z^3)\,, \qquad \psi \to \psi' \tag{3.6}$$

for new α', ϕ' and ψ'.

[1] In ref. [12] the constant α is required to vanish by the Bianchi identity. However, we do not agree with this result and leave α non-vanishing.

We now require that the expression (3.3) and that with the replacements (3.6) are compatible each other. Let us first consider G_{123}. From the expression (3.3) we have $G_{123} = 0$. From the other expression we have $G_{123} = \frac{1}{6}\partial_3^2\phi' Z$, which is obtained from $G_{\bar{1}\bar{2}3}$ in eq. (3.3) by the replacements (3.6). Thus we obtain a condition

$$G_{123}: \quad \partial_3^2\phi' = 0\,. \tag{3.7}$$

Similarly, we obtain conditions

$$\begin{aligned}
G_{2\bar{2}1} + G_{3\bar{3}1}: &\quad \partial_{\bar{2}}\partial_3\phi' = 0\,,\\
G_{1\bar{1}2} + G_{3\bar{3}2}: &\quad \partial_{\bar{1}}\partial_3\phi' = 0\,,\\
G_{\bar{1}\bar{2}3}: &\quad \partial_3^2\phi = 0\,,\\
G_{\bar{2}2\bar{1}} + G_{3\bar{3}\bar{1}}: &\quad \partial_2\partial_3\phi = 0\,,\\
G_{\bar{1}1\bar{2}} + G_{3\bar{3}\bar{2}}: &\quad \partial_1\partial_3\phi = 0\,,\\
G_{1\bar{2}\bar{3}}: &\quad \partial_1^2\phi = \partial_{\bar{2}}^2\phi'\,,\\
G_{\bar{1}2\bar{3}}: &\quad \partial_2^2\phi = \partial_{\bar{1}}^2\phi'\,.
\end{aligned} \tag{3.8}$$

The component $G_{1\bar{1}3} + G_{2\bar{2}3}$ vanishes in both of the two expressions and gives no condition. These conditions fix the forms of ϕ and ϕ' as

$$\begin{aligned}
\phi &= m_1(z^1)^2 + m_2(z^2)^2 + 2az^1z^2 + b_1z^1 + b_2z^2 + b_3z^3\,,\\
\phi' &= m_2(\bar{z}^1)^2 + m_1(\bar{z}^2)^2 + 2a'\bar{z}^1\bar{z}^2 + b_1'\bar{z}^1 + b_2'\bar{z}^2 + b_3'z^3\,,
\end{aligned} \tag{3.9}$$

where m_1, m_2, a, a', b_i and b_i' are arbitrary constants. We further obtain conditions

$$\begin{aligned}
G_{\bar{1}23}: &\quad \partial_{\bar{1}}^2\psi = \partial_2^2\psi'\,,\\
G_{1\bar{2}3}: &\quad \partial_{\bar{2}}^2\psi = \partial_1^2\psi'\,,\\
G_{31\bar{1}}: &\quad \partial_{\bar{1}}\partial_{\bar{2}}\psi = -\partial_1\partial_2\psi'\,, \qquad a = -a'\,.
\end{aligned} \tag{3.10}$$

By a linear transformation $z^i \to U^i{}_j z^j$ $(i,j = 1,2)$ with a unitary matrix U we can set $a = -a' = 0$.

So far we have not used a particular form of Z. We now examine the remaining conditions first by using the asymptotic form $Z \sim R^4/r^4$ for $r \to \infty$ to fix the coefficients in eq. (3.9) and α, α'. We then check that the conditions are satisfied also for $r < \infty$. From the equation for $G_{1\bar{1}\bar{3}}$ we obtain

$$\begin{aligned}
G_{1\bar{1}\bar{3}}: \quad & -\frac{1}{6}\partial_1\partial_2\phi Z + \frac{1}{12}\left(\alpha\partial_{\bar{3}}Z + 2\partial_1\phi\partial_2 Z - 2\partial_2\phi\partial_1 Z\right) =\\
&= \frac{1}{6}\partial_{\bar{1}}\partial_{\bar{2}}\phi' Z - \frac{1}{12}\left(\alpha'\partial_{\bar{3}}Z + 2\partial_{\bar{1}}\phi'\partial_{\bar{2}}Z - 2\partial_{\bar{2}}\phi'\partial_{\bar{1}}Z\right).
\end{aligned} \tag{3.11}$$

The equation for $G_{2\bar{2}\bar{3}}$ gives the same condition. Substituting the asymptotic form $Z \sim R^4/r^4$ and eq. (3.9) into eq. (3.11) we find $\alpha' = -\alpha$ and $b_1 = b_2 = b_1' = b_2' = 0$. The remaining conditions become

$$
\begin{aligned}
G_{1\bar{1}\bar{2}}: &\qquad \partial_1\partial_{\bar{3}}\psi' = \frac{1}{12}(\alpha\partial_{\bar{2}} + 2b_3\partial_1)Z\,,\\
G_{3\bar{3}\bar{1}}: &\qquad \partial_2\partial_{\bar{3}}\psi' = -\frac{1}{12}(\alpha\partial_{\bar{1}} - 2b_3\partial_2)Z\,,\\
G_{\bar{1}\bar{2}\bar{3}}: &\qquad \partial_{\bar{3}}^2\psi' = \frac{1}{6}b_3\partial_{\bar{3}}Z\,,\\
G_{23\bar{3}}: &\qquad \partial_{\bar{1}}\partial_{\bar{3}}\psi = -\frac{1}{12}(\alpha\partial_2 - 2b_3'\partial_{\bar{1}})Z\,,\\
G_{12\bar{2}}: &\qquad \partial_{\bar{2}}\partial_{\bar{3}}\psi = \frac{1}{12}(\alpha\partial_1 + 2b_3'\partial_{\bar{2}})Z\,,\\
G_{12\bar{3}}: &\qquad \partial_{\bar{3}}^2\psi = \frac{1}{6}b_3'\partial_{\bar{3}}Z\,.
\end{aligned}
\tag{3.12}
$$

Comparing the equation obtained by applying $\partial_{\bar{3}}$ to the first equation in eq. (3.12) and that obtained by applying ∂_1 to the third equation we find $\alpha = 0$. Then, eq. (3.12) determines ψ, ψ' as

$$
\begin{aligned}
\partial_{\bar{3}}\psi &= \frac{1}{6}b_3'Z + f(z^1, z^2, z^3)\,,\\
\partial_{\bar{3}}\psi' &= \frac{1}{6}b_3Z + f'(\bar{z}^1, \bar{z}^2, z^3)\,,
\end{aligned}
\tag{3.13}
$$

where f and f' are arbitrary functions of each variables. Substituting eq. (3.13) into the $\bar{z}^3$ derivative of eq. (3.10) and using the asymptotic form $Z \sim R^4/r^4$ we obtain $b_3 = b_3' = 0$.

As a result of these analyses at asymptotic region $r \sim \infty$ we obtain

$$
\begin{aligned}
\phi &= m_1(z^1)^2 + m_2(z^2)^2\,,\\
\phi' &= m_2(\bar{z}^1)^2 + m_1(\bar{z}^2)^2\,.
\end{aligned}
\tag{3.14}
$$

We have to check that eqs. (3.10), (3.11) and (3.12) are satisfied even for $r < \infty$. Substituting eq. (3.14) into eq. (3.12) we find that their right-hand sides vanish. The general solution of these equations are

$$
\begin{aligned}
\psi &= f(z^1, z^2, z^3)\bar{z}^3 + g(z^1, \bar{z}^1, z^2, \bar{z}^2, z^3)\,,\\
\psi' &= f'(\bar{z}^1, \bar{z}^2, z^3)\bar{z}^3 + g'(z^1, \bar{z}^1, z^2, \bar{z}^2, z^3)\,,
\end{aligned}
\tag{3.15}
$$

where f, f', g and g' are arbitrary functions of each variables. The conditions in eq. (3.10) then require

$$
\partial_{\bar{1}}^2 g = \partial_2^2 g'\,, \qquad \partial_{\bar{2}}^2 g = \partial_1^2 g'\,, \qquad \partial_{\bar{1}}\partial_{\bar{2}} g = -\partial_1\partial_2 g'\,. \tag{3.16}
$$

The conditions that ψ and ψ' in eq. (3.15) are harmonic are

$$
\begin{aligned}
\partial^2 g(z^1, \bar{z}^1, z^2, \bar{z}^2, z^3) &= -\partial_3 f(z^1, z^2, z^3)\,,\\
\partial^2 g'(z^1, \bar{z}^1, z^2, \bar{z}^2, z^3) &= -\partial_3 f'(\bar{z}^1, \bar{z}^2, z^3)\,.
\end{aligned}
\tag{3.17}
$$

The functions f and f' do not appear in G_{mnp} as one can see by substituting eq. (3.15) into eq. (3.3). We only need to consider g and g'. Eq. (3.17) means that $\partial^2 g$ and $\partial^2 g'$ are

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independent of $\bar{z}^1, \bar{z}^2$ and z^1, z^2 respectively. These conditions are automatically satisfied when g and g' satisfy eq. (3.16). The functions g and g' need not be harmonic.

Finally, we have to consider eq. (3.11). Substituting eq. (3.14) into eq. (3.11) we obtain

$$\left(m_1 z^1 \partial_2 - m_2 z^2 \partial_1 + m_2 \bar{z}^1 \partial_{\bar{2}} - m_1 \bar{z}^2 \partial_{\bar{1}}\right) Z = 0 \,. \tag{3.18}$$

This means that Z is invariant under SO(2) rotation of $(\sqrt{m_1}\, z^1, \sqrt{m_2}\, z^2)$ and $(\sqrt{m_2}\, \bar{z}^1, \sqrt{m_1}\, \bar{z}^2)$. Therefore, Z must be a function of SO(2) invariant variables $r^2 = 2\left(z^1\bar{z}^1 + z^2\bar{z}^2\right)$, $m_1(z^1)^2 + m_2(z^2)^2$, $m_2(\bar{z}^1)^2 + m_1(\bar{z}^2)^2$ and $m_1 z^1 \bar{z}^2 - m_2 z^2 \bar{z}^1$.

Let us summarize the result. The general form of the three-form flux G_{mnp} which preserves the $\mathcal{N} = 2$ supersymmetry at the first order of the perturbation is given by eq. (3.3) with $\alpha = 0$, ϕ in eq. (3.14) and ψ replaced by $g(z^1, \bar{z}^1, z^2, \bar{z}^2, z^3)$ satisfying eq. (3.16) for some function $g'(z^1, \bar{z}^1, z^2, \bar{z}^2, z^3)$. Thus, ϕ, which is an arbitrary holomorphic function in the $\mathcal{N} = 1$ case [12], is severely restricted to a quadratic function in the $\mathcal{N} = 2$ case. Such $\mathcal{N} = 2$ preserving perturbation is possible only when the warp factor Z satisfies eq. (3.18).

In our analysis at the first order of the perturbation we did not need the condition $m_1 = m_2$ to obtain the $\mathcal{N} = 2$ supersymmetry. At higher orders [20] we would need the condition $m_1 = m_2$ since these parameters correspond to masses of two $\mathcal{N} = 1$ chiral multiplets, which should be combined into an $\mathcal{N} = 2$ hypermultiplet. This is indeed the case in the field theory side. To see this let us consider two $\mathcal{N} = 1$ chiral supermultiplets (A_1, ψ_1) and (A_2, ψ_2), where A_1, A_2 are complex scalar fields and ψ_1, ψ_2 are Weyl spinor fields, with the action

$$\begin{aligned} S = \int d^4x \Big[& -\partial_\mu A_1^* \partial^\mu A_1 - \partial_\mu A_2^* \partial^\mu A_2 - i\psi_1 \sigma^\mu \partial_\mu \bar{\psi}_1 - i\psi_2 \sigma^\mu \partial_\mu \bar{\psi}_2 - \\ & - m_1^2 A_1^* A_1 - m_2^2 A_2^* A_2 - \frac{1}{2} m_1 \left(\psi_1\psi_1 + \bar{\psi}_1\bar{\psi}_1\right) - \frac{1}{2} m_2 \left(\psi_2\psi_2 + \bar{\psi}_2\bar{\psi}_2\right) \Big] \end{aligned} \tag{3.19}$$

Here we have used the two-component spinor notation in ref. [21]. S is invariant under the $\mathcal{N} = 1$ supertransformation

$$\delta A_i = \sqrt{2}\epsilon\psi_i \,, \qquad \delta\psi_i = \sqrt{2} i \sigma^\mu \bar{\epsilon} \partial_\mu A_i - \sqrt{2} m_i \epsilon A_i^* \qquad (i = 1, 2) \,. \tag{3.20}$$

The exact $\mathcal{N} = 2$ supersymmetry of course requires $m_1 = m_2$. However, even for $m_1 \neq m_2$, it is also invariant under the second supertransformation

$$\begin{aligned} \delta A_1 &= \sqrt{2}\epsilon\psi_2 \,, & \delta\psi_1 &= \sqrt{2} i \sigma^\mu \bar{\epsilon} \partial_\mu A_2 - \sqrt{2} m_1 \epsilon A_2^* \,, \\ \delta A_2 &= -\sqrt{2}\epsilon\psi_1 \,, & \delta\psi_2 &= -\sqrt{2} i \sigma^\mu \bar{\epsilon} \partial_\mu A_1 + \sqrt{2} m_2 \epsilon A_1^* \end{aligned} \tag{3.21}$$

at the first order in m_1, m_2. Thus, the condition $m_1 = m_2$ is needed only in quadratic and higher order terms for the $\mathcal{N} = 2$ supersymmetry.

Acknowledgments

One of the authors (M.N.) would like to thank N. Constable, D.Z. Freedman, A. Hanany, W. Taylor, J. Troost and A. Tsuchiya for helpful discussions, and the Center for Theoretical Physics of MIT for hospitality. This work is supported in part by the Nishina Memorial Foundation and by funds provided by the U.S. Department of Energy (D.O.E.) under cooperative research agreement #DF-FC02-94ER40818.

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